ANALYSIS OF ONLINE OVERLAPPING AUCTIONS: 
A MANAGERIAL PERSPECTIVE

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An interesting aspect of current online auctions is that often multiple mechanisms exist concurrently or in a series for buying and selling of an identical item and frequently these mechanisms are offered by the same website. Figure 1 illustrates an instance of multiple overlapping online auctions at Samsclub.com.

It is hard to view today’s web-based online auction market as being independent and one that operates in an isolated environment. Using search engines and web-links that are commonly provided by auctioneers, a bidder can easily access multiple concurrent auctions for the same good. Since multiple sources are visible and reachable without high cost, there exists interdependency and interplay among multiple concurrent auctions, which might influence the outcome of each individual auction. Using the formalization of overlapping auctions given in (Bapna et al., 2005), we are motivated to analyze these interesting market phenomena in the context of designing optimal multiple overlapping online auctions.

We develop a model to characterize bidder arrival process, their splitting into available overlapping auctions, and the price formation processes. The theoretical properties derived from our model are empirically tested using a dataset from Sam’s Club online auction site. Based on the results, we determine control factors such as ‘degree of overlap’ among the auctions and ‘duration’ of each auction which together form the optimal market design strategy.

When two auctions share certain period of time space, we say that the two auctions are overlapping with each other. In such case, the auction which has earlier (later) closing time is the preceding (succeeding) overlapping auction. When there are $N$ bidders and each bidder’s valuation of the object (or item) is $i.i.d.$ uniform random variable over the interval $[\mu - s, \mu + s]$, then the expected price in an English auction is

$$\text{Price in auction } i = \mu + s - \frac{4 \cdot s}{N_i + 1}$$

(1)

When there are multiple overlapping auctions, bidders arrival stream splits into multiple available auctions thereby decreasing the arrival intensity of any given auction. Due to the
decreased intensity and hence reduced competition, the expected price is likely to be lower. As $N_i$ is a function of the arrival intensity, the reduced arrival intensity will decrease the expected revenue. We first derive the following propositions and then empirically examine the implications using the dataset we collected from Samsclub.com.

**Proposition 1.** As the duration of auction $i$ increases, the revenue increases.

From equation (1), as the duration increases, an auction can accommodate more bidders giving higher expected revenue.

**Proposition 2.** The overlap has negative impact on a given auction price and the price decrease increases as the overlapped time increases.

As the overlapped time increases, the number of bidders in the course of an auction decreases and the decreased competition places a downward pressure on the revenue.

We validate the implications using the dataset from Samsclub.com. The dataset includes 932 auctions for various electronic goods. Table 1 shows that the price of an auction decreases as the overlapped time increases. Number of bidders in each auction and duration have positive impact on a given auction price. All the results and the significance level confirm our propositions.

$$\text{Price/Max} = \beta_0 + \beta_1 \cdot \text{Number of Bidders} + \beta_2 \cdot OTP + \beta_3 \cdot OTS + \beta_4 \cdot \text{Duration}$$

(2)

<table>
<thead>
<tr>
<th></th>
<th>Coef.</th>
<th>S. E.</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Bidders</td>
<td>0.004</td>
<td>0.001</td>
<td>0.00</td>
</tr>
<tr>
<td>OTP (Overlapped time</td>
<td>-0.010</td>
<td>0.002</td>
<td>0.00</td>
</tr>
<tr>
<td>OTS (Overlapped time</td>
<td>-0.005</td>
<td>0.002</td>
<td>0.03</td>
</tr>
<tr>
<td>Duration</td>
<td>0.057</td>
<td>0.014</td>
<td>0.00</td>
</tr>
<tr>
<td>Constant</td>
<td>0.726</td>
<td>0.018</td>
<td>0.00</td>
</tr>
</tbody>
</table>

R-Sq = 12.0%  R-Sq(adj) = 11.6%

Using constant arrival intensity $\lambda$ and intensity weakening factors $\alpha$ and $\gamma$ for preceding and succeeding overlapping auctions during OTP and OTS respectively, we formulate the seller’s revenue maximizing problem as follows.

$$\text{Max} \sum_{i=1}^{q} \left( \mu + s - \frac{4 \cdot s}{\lambda \cdot d_i - \alpha \cdot OTP_i - \gamma \cdot OTS_i + 1} \right)$$

s.t. $d_i \leq T$

$\lambda \cdot d_i - \alpha \cdot OTP_i - \gamma \cdot OTS_i \geq 1$

$\alpha, \gamma, \lambda \geq 0$

Note that, as in figure 2, when the seller has a fixed time horizon $T$ in which $q$ auctions are offered, as the duration of auctions increases, the total overlapped time increases exponentially.
As the duration increases, a given auction experiences more overlapping auctions than just immediately preceding and succeeding auctions. Figure 2 shows the behavior of average OTP, OTS and corresponding approximation using a continuous function. We define $d_p$ and $d_s$ as

$$d_p : \text{Average OTP per auction} = \frac{1}{q} \cdot \sum_{i=1}^{q} OTP_i$$

$$d_s : \text{Average OTS per auction} = \frac{1}{q} \cdot \sum_{i=1}^{q} OTS_i$$

We rewrite (3) using $d_p$ and $d_s$. Note $d_p$ and $d_s$ are approximated with functions of duration $d$.

$$\text{Max } q \left( \mu + s - \frac{4 \cdot s}{\lambda \cdot d - \alpha \cdot d_p - \gamma \cdot d_s + 1} \right)$$

s.t. $d \leq T$

$$\lambda \cdot d - \alpha \cdot d_p - \gamma \cdot d_s \geq 1$$

$$\alpha, \gamma, \lambda \geq 0$$

We obtain the optimal duration and the optimal degree of overlap to derive the following propositions.

$$d^* = \log \left( \frac{\lambda}{\alpha \cdot c_p + \gamma \cdot c_s} \right), d_p^* = \frac{c_p \cdot \lambda}{\alpha \cdot c_p + \gamma \cdot c_s} \text{ and } d_s^* = \frac{c_s \cdot \lambda}{\alpha \cdot c_p + \gamma \cdot c_s}$$

**Proposition 3.** As the arrival intensity increases the optimal degree of overlap increases.

While the increased auction duration will increase OTP and OTS which will weaken the arrival stream into a given auction, increased $\lambda$ mitigates the loss of bidders and the seller can increase the degree of overlap as well as duration.

**Proposition 4.** The arrival rate weakening factors $\alpha$ and $\gamma$ are negatively correlated with the optimal duration.

As $\alpha$ and $\gamma$ increases, the optimal strategy is to decrease the degree of overlap so as to minimize the loss due to the overlap.

Currently we are conducting simulation studies to find optimal solutions for empirically calibrated $\mu, s, \lambda, \alpha$ and $\gamma$. As a part of our study we are working on multi-unit auctions as well. Suppose a seller has $q$ units of goods. A question that arises in such a case is: Is one $q$-unit
auction optimal, are \( q \) single-unit auctions optimal or some mixture of single-unit and multi-unit auctions optimal? How is the optimal degree of overlap related to the lot-size?

References